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Linac, Beam Dynamics

Rough Estimation of Effects of Fast-Changing Stray Field in Long Transport of RTML

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Rough estimation of effects of fast changing stray field in long transport of RTML
- Emittance dilution in Turnaround

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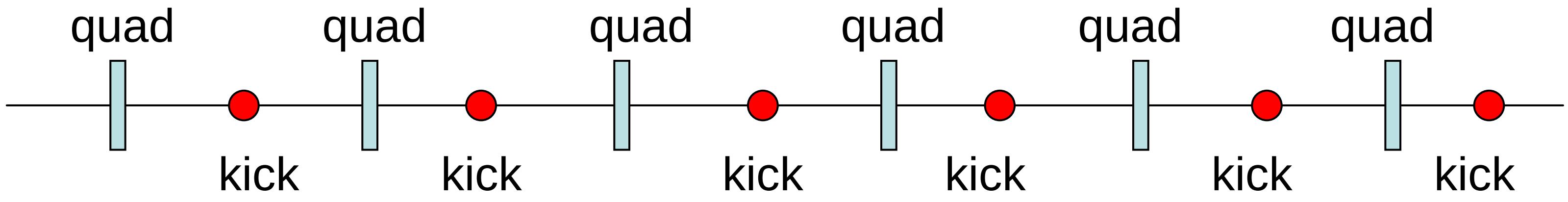
Long Transport in RTML

- DR may be located at the center of ILC site.
- Need to transport low emittance, 5GeV beam. The length is approximately the length of the main linac.
- Stray field will affect the beam orbit.
- Orbit after turnaround will be corrected by feed-forward.
- Orbit in the turnaround will increase emittance.
- Emittance increase in the long transport line itself is not studied here.

Assumptions and Approximations

- Straight beam line
- FODO lattice
- Kick by the stray fields between two consecutive quads are represented by a kick at one position, the center of the section.
- The stray field strength (integrated between two quads) is independent and random, and the RMS of the strength is constant along the beam line.
- There are many quads (Sections between them) and the effects can be treated statistically.



kick angle at i-th section: $\theta_i = l\dfrac{cB_i}{E}$

l : Length Between Quads, c : Speed of Light

B_i : Average of transverse (magnetic) field strength in the i-th section

$$B_i = \int B(s)ds / l$$

(B : magnetic field strength, integrate between quads, l : quad spacing)

E : Beam energy

Position change at the end of the beam line: $y = \sum_i \theta_i \sqrt{\beta\beta_i}\sin\varphi_i$

β : betafunction at the end, β_i : betafunction at i-th section,

φ_i : phase advance from i-th section to the end.

$\sqrt{\beta\beta_i}\sin\varphi_i$ is R_{12} from i-the section to the end of the line

RMS of position change at the end of the beam line:

$$\sqrt{\left\langle y^2 \right\rangle} = \sqrt{\left\langle \left[\sum_i \theta_i \sqrt{\beta \beta_i} \sin \varphi_i \right]^2 \right\rangle} \approx \sqrt{\beta \sum_i \left\langle \theta_i^2 \right\rangle \beta_i \sin^2 \varphi_i} \approx \frac{c}{E} \sqrt{\frac{L l \beta \overline{\beta}}{2}} B_{RMS}$$

$\left\langle \; \right\rangle$: average over ensemble, B_{RMS} : RMS of average of field

$L = l \times N_q$: Length of the beam line.

N_q : Number of quads

Here, we used

$$\left\langle \theta_i \theta_j \right\rangle = \begin{cases} 0 & (i \neq j) \\ \left\langle \theta_i^2 \right\rangle = \left(\dfrac{l c B_{RMS}}{E} \right)^2 & (i = j) \end{cases} ,$$

$\sin^2 \varphi_i \to 1/2$, $\beta_i \to \overline{\beta}$ (averages over many sections), then,

$$\sum_{\mathrm{i}} \left\langle \theta_i^2 \right\rangle \beta_i \sin^2 \varphi_i \to N_q \left(\frac{l c B_{RMS}}{E} \right)^2 \overline{\beta} \frac{1}{2}$$

If we Require the beam orbit jitter smaller than 10% of beam size,

$$\sqrt{\left\langle y^2 \right\rangle} < 0.1\,\sigma_y = 0.1\sqrt{\varepsilon\beta}, \quad B_{RMS} < 0.1\frac{E}{c}\sqrt{\frac{2\varepsilon}{Ll\bar{\beta}}} \quad (\varepsilon : \text{emittance}).$$

Using parameters;

$$\varepsilon = 2\times10^{-12}\ \text{m}, \quad E = 5\times10^{9}\ \text{eV}$$

$$L \approx 1\times10^{4}\ \text{m}, \quad l \approx 35\ \text{m}, \quad \bar{\beta} \approx 100\ \text{m (phase advance 45 deg./FODO cell)},$$

$$B_{RMS} < 1.0\times10^{-9}\ \text{T.} \quad \text{(without feed-forward.)}$$

This limit is inversely proportional to l (half length of FODO cell, note $\bar{\beta} \propto l$) and $\sqrt{L}$ (L is total length of the beam line).

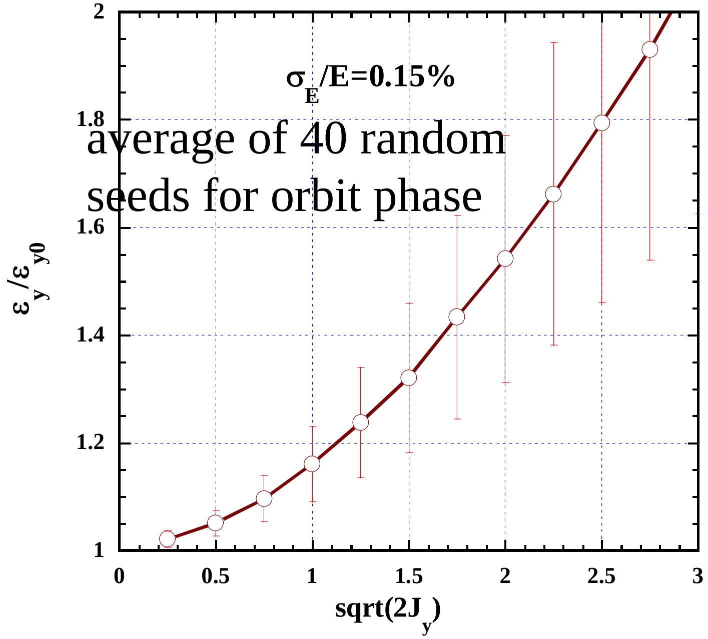
If we rely on the feed-forward in the turnaround after the long transport line, this limit is not relevant.

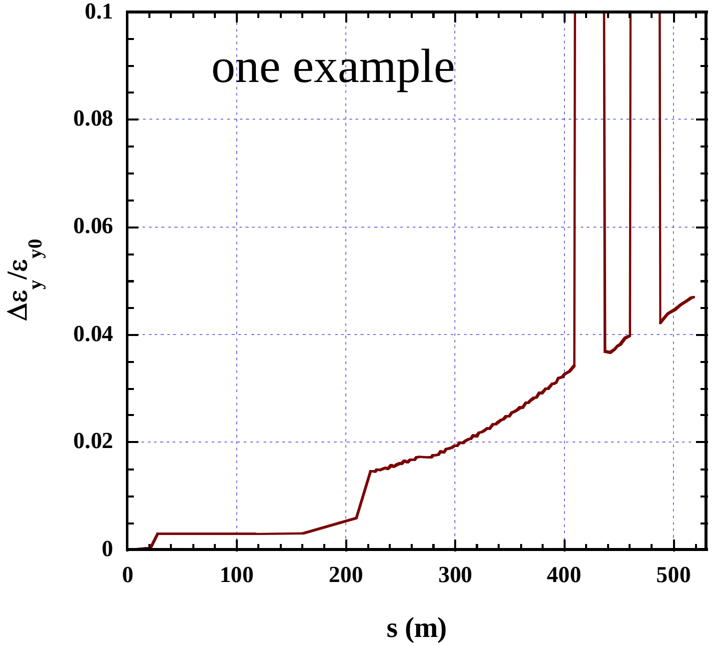
The limit will be from the emittance dilution due to dispersive effect in the turnaround caused by the orbit fluctuations.

Tracking simulation in the turnaround (RTML before BC1), using SAD

Set orbit at the beginning of present version of RTML and looked at emittance at the entrance of Bunch Compressor, after the turnaround and spin rotator.

Initial $\gamma\varepsilon$ = 2E-8 m, σ_E/E =0.15%, 4000 macro-particles.





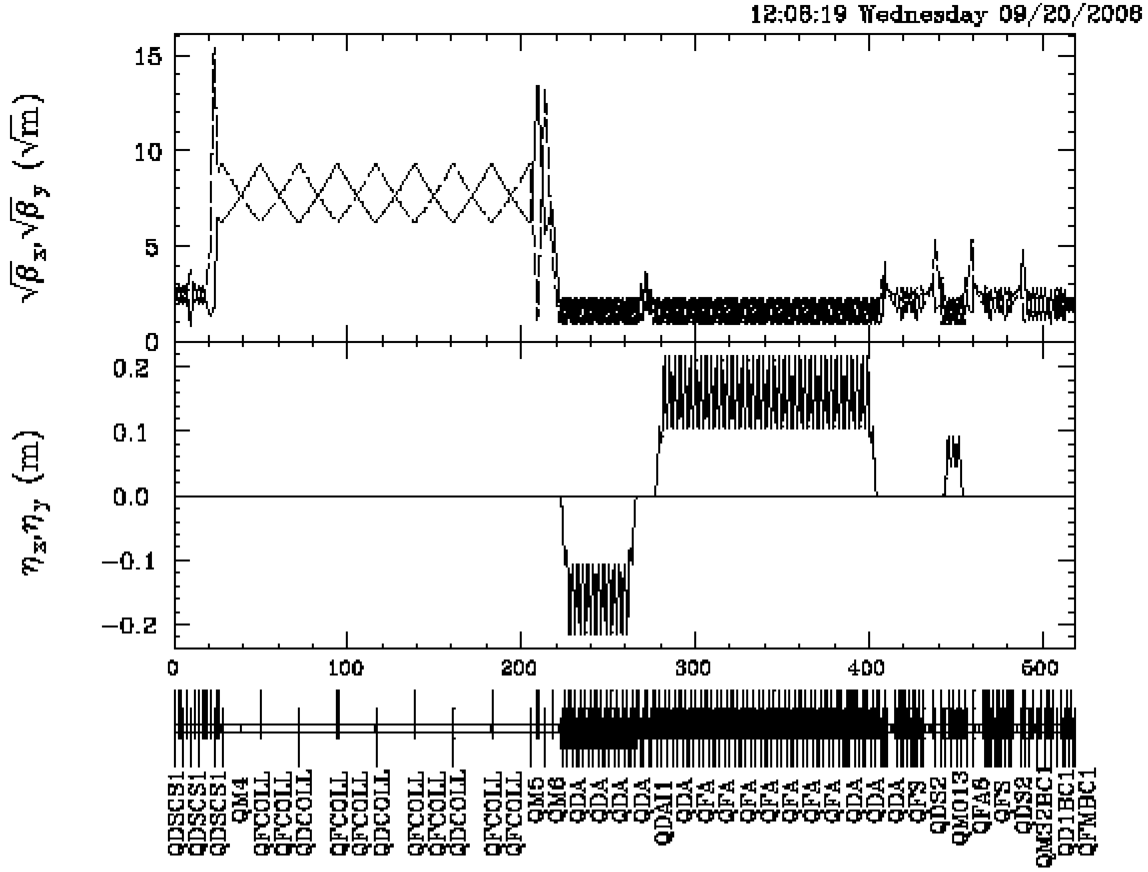
0.75-sigma jitter will be tolerable, then, requirement is

$$B_{RMS} < 7.5\times10^{-9}\ \mathrm{T.}\ \text{(with feed-forward)}$$

Present RTML optics, up to BC entrance,

http://www.linearcollider.org/wiki/doku.php?id=rdr:rdr_as:rtml_lattice

Translated to SAD format by Shi-Lun Pei.



Summary

Tolerable orbit jitter in the turn-around is about 0.75-sigma, which increase emittance about 5% in the region of the turnaround, considering orbit feed-forward.

Then, requirement of stray field in the long (~10 km) straight section of the RTML will be

$$B_{RMS} < 7.5 \times 10^{-9} \text{ T.}$$

where B_{RMS} is the RMS of average stray field strength between two quads,

$$\int B_x(s)ds / l \qquad (B_x : \text{horizontal magnetic field, } l : \text{quad spacing})$$

We assumed this average is independent and random for each section.

This limit is relevant for fields which change faster than orbit feedback, either inter-pulse or intra-pulse.