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Effect of long-range wakefiled of accelerating cavities in the ILC main linacs are roughly estimated. First, assuming each bunch is rigid, dilution of multibunch-projected emittance is estimated using results of the references [1] [2]. Then, single bunch emittance dilution induced by the long range wakefield is considered.

I. RIGID BUNCH

A. tolerance of 'rms of sum-wake'

Assuming each bunch is rigid, multibunch projected emittance dilution is evaluated from distribution of position and angle of each bunch as

$$\Delta\epsilon = \gamma(\overline{x^2} - \bar{x}^2) + 2\alpha(\overline{xx'} - \bar{x}\overline{x'}) + \beta(\overline{x'^2} - \overline{x'}^2), \tag{1}$$

where $\bar{}$ denotes average over bunches, x and x' are position and angle of each bunch, α, β, γ Twiss-parameters. For example,

$$\bar{x} = \sum_{m=1}^{M} q_m x_m / \sum_{m=1}^{M} q_m, \tag{2}$$

where M is total number of bunches, x_m position of m-th bunch and q_m charge of m-th bunch.

If we assume bunch to bunch transverse position difference (orbit change due to the wakefield) are much less than the typical misalignment of cavities, expected multibunch projected emittance dilution due to long-range wakefield in a high energy linac is expressed as

$$\langle\Delta\epsilon\rangle = \frac{e^2 L^2}{2E_f} \sum_i \frac{a_i^2 \beta_i S_{i,\mathrm{rms}}^2}{E_i}, \tag{3}$$

where $\langle\rangle$ denotes overage over many ensembles [1] [2]. L is the length of each cavity, E_f the final beam energy, E_i the beam energy at i-th cavity, β_i the beta-function at i-th cavity, $\sum_i$ denotes sum of all cavities. a_i is the rms of misalignment (offset) of i-th cavity, assuming every cavity has independently random misalignment. Assuming bunch spacing is uniform, $S_{i,\mathrm{rms}}$, 'rms of sum-wake' of i-th cavity is defined as

$$S_{i,\mathrm{rms}} \equiv \sum_{m=1}^{M} q_m (S_{i,m} - S_{i,a})^2 / \sum_{m=1}^{M} q_m, \tag{4}$$

where 'sum-wake' of i-th cavity is defined as

$$S_{i,m} \equiv \sum_{k=1}^{m-1} q_k W_i(k), \tag{5}$$

and 'average of sum-wake' as

$$S_{i,a} \equiv \sum_{m=1}^{M} q_m S_{i,m} / \sum_{m=1}^{M} q_m, \tag{6}$$

where $W_i(k)$ is the wakefunction (per length) of the i-th cavity between k times of bunch spacing, q_m the charge of m-th bunch.

Let us assume that misalignment of the all cavities have the same rms, a, and rms of 'sum-wake' of the all cavities are the same, $S_{i,\mathrm{rms}} = S_{\mathrm{rms}}$. Then, Eq. (3) becomes

$$\langle\Delta\epsilon\rangle = \frac{e^2 L^2 a^2 S_{\mathrm{rms}}^2}{2E_f} \sum_i \frac{\beta_i}{E_i}, \tag{7}$$

here, $\sum_i \beta_i/E_i$ can be calculated from the optics of the linac.

TABLE I: Main Linac parameters for rough estimations.

Accelerating gradient	35 MV/m
Beam energy of the first section	5 GeV → 125 GeV
Beam energy of the second section	125 GeV → 250 GeV
Length of FODO cell of the first section	60 m
Length of FODO cell of the second section	90 m
Phase advance per FODO cell	60 degree
Length of accelerating cavity	1.036 m
Number of particles per bunch	2×10^{10}

Because the length of the cavity is short compared with the beta-function, the summation can be expressed as integral as

$$\sum_i \frac{\beta_i}{E_i} \to \int_{E_0}^{E_f} \frac{\beta}{E} \frac{dE}{gL}, \tag{8}$$

where g is the accelerating gradient. For the FODO optics shown in the Table I,

$$\int_{E_0}^{E_f} \frac{\beta}{E} dE = \int_{E_0}^{E_1} \frac{\beta}{E} dE + \int_{E_1}^{E_f} \frac{\beta}{E} dE = \bar{\beta}_1 \log{(E_1/E_0)} + \bar{\beta}_2 \log{(E_f/E_1)}, \tag{9}$$

where the first(second) term is from the first(second) section. $E_1 = 125$ GeV is the beam energy at the transition from the first section to the second section. $\bar{\beta}_{1(2)}$ is the average of the beta function in the first(second) section. Since the phase advance is $\pi/3$ per FODO cell, from thin lens approximation,

$$\bar{\beta}_{1(2)} = \frac{11}{6\sqrt{3}} l_{1(2)} \approx l_{1(2)}, \tag{10}$$

where $l_{1(2)}$ is the length of FODO cell in the first(second) section. Then,

$$\sum_i \frac{\beta_i}{E_i} \approx (\log{(E_1/E_0)} l_1 + \log{(E_2/E_1)} l_2) \frac{1}{gL}. \tag{11}$$

Dilution of normalized emittance is

$$\langle \Delta(\gamma\epsilon) \rangle \approx \frac{e^2 L a^2 S_{\mathrm{rms}}^2}{2 m_e c^2 g} (\log{(E_1/E_0)} l_1 + \log{(E_2/E_1)} l_2), \tag{12}$$

where m_e is electron mass.

Using parameters in the Table I,

$$\langle \Delta(\gamma\epsilon) \rangle [m] \approx 7.4 \times 10^{-12} \times (a[m])^2 \left(S_{\mathrm{rms}}[V/m^2]\right)^2. \tag{13}$$

If the rms of cavity misalignment is 300 μm,

$$\langle \Delta(\gamma\epsilon) \rangle [m] \approx 6.7 \times 10^{-19} \times \left(S_{\mathrm{rms}}[V/m^2]\right)^2. \tag{14}$$

Allowing emittance dilution by 10% of nominal ($\gamma\epsilon_0 = 2 \times 10^{-8} m$), tolerable ‘rms of sum-wake’ is

$$S_{\mathrm{rms}} < 5.5 \times 10^4 [V/m^2], \tag{15}$$

which is proportional to squre root of the tolerable dilution. If the wakefunction is known, ‘rms of sum-wake’ is calculated as Eqs. from (4) to (6) and we can check if the wakefield is tolerable.

B. Approximate estimation of ‘rms of sum-wake’

In principle, description in the previous section is enough to judge whether the given long-range wakefunction is tolerable or not. But it may be useful to have some ‘feeling’ for tolerable wakefunctions. Note that the estimations in this section are based on very rough approximations.

1. *No damping*

First, let us estimate 'rms of sum-wake' approximately assuming no damping. Let A be the typical amplitude of the wakefunction. Then,

$$S_{i,m} = q \sum_{k=1}^{m-1} W(kt_b), \tag{16}$$

$$S_{\rm rms}^2 \approx \frac{1}{M} q^2 \sum_{m=1}^{M} \sum_{k=1}^{m-1} W(kt_b) \sum_{j=1}^{m-1} W(jt_b) - \left(\frac{1}{M} q \sum_{m=1}^{M} \sum_{k=1}^{m-1} W(kt_b) \right)^2 \approx \frac{M}{12} q^2 A^2. \tag{17}$$

We assumed all bunches has the same charge q and used $W(kt_b)W(jt_b) \to \delta_{kj} A^2/2$.

Using $q = 2 \times 10^{10} e$ and Eq. (15), requirement for A becomes,

$$A < 6.0 \times 10^{13}/\sqrt{M} \; [V/C/m^2]. \tag{18}$$

Note that the requirement becomes severe for large M, total number of bunches, because damping of the wakefiled is ignored here.

2. *With damping*

Next, let us estimate 'rms of sum-wake' approximately with damping. Let A_0 be the typical amplitude of the wakefunction (before damping)and α the typical damping constant. Then, the wakefunction at time t can be expressed as

$$|W(t)| \approx A_0 \exp{(-\alpha t)}. \tag{19}$$

Assuming all bunches have the same charge q,

$$|S_{i,m}| \approx q A_0 \sum_{k=1}^{m-1} \sin(\theta_k) \exp{(-k \alpha t_b)}. \tag{20}$$

Let us assume the phase of the wakefield (θ_k) has no correlation with the bunch spacing, then, $\sin(\theta_k)\sin(\theta_j) \to \delta_{kj}/2$. Also assuming the damping between one bunch spacing is small, $\alpha t_b << 1$, but the number of bunch is large, $\exp{(-M \alpha t_b)} << 1$, after some manipulations the 'rms of sum-wake' becomes,

$$S_{\rm rms} \approx \frac{q A_0}{\sqrt{2M}} \frac{1}{2 \alpha t_b}. \tag{21}$$

Note that the requirement becomes loose for large M, total number of bunches, because we assume the wakefield induced by the first bunch is completely damped before it reaches the last bunch. If M is very large, all bunches except in the head part of a bunch train feel the same wakefield induced by previous bunches. (Also note that we assume typical misalignment of cavities is much larger than the orbit change due to wakefield.) Consequently, these bunches have the same orbit and rms of the orbit difference from the average orbit becomes small. This was pointed out, for example, in the reference [3].

II. SINGLE BUNCH EFFECT DUE TO LONG-RANGE WAKEFIELD

Though bunch length can be assumed to be short so that the long-range wakefield is almost constant within a bunch, dispersive effects will be a source of emittance dilution because of finite momentum spread in each bunch. In

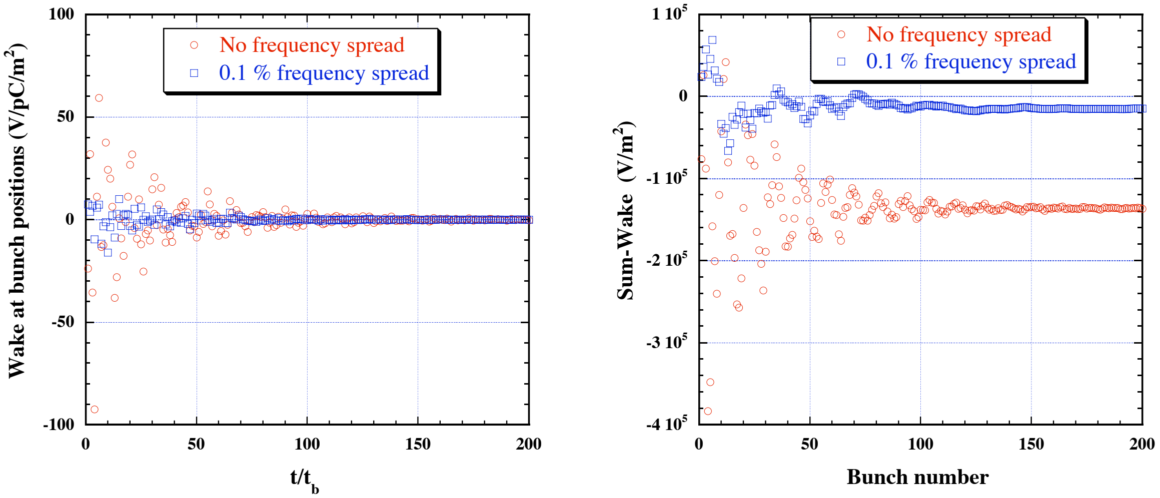

FIG. 1: Long-range wakefunction at bunch positions (left),nd 'sum-wake', calculated from HOM in TESLA-TDR.[3].

the reference [4], we estimated tolerance of fast movement of cavity tilt angle as 2 μrad, assuming every cavity has independent, random tilt, where accelerating gradient was 35 MV/m. It means tolerance of transverse kick at each cavity is

$$V_{T,\mathrm{tol}} = 35\mathrm{MV/m} \times 2 \times 10^{-6} \times L = 70\mathrm{V}, \tag{22}$$

where L is the length of a cavity. Long-range wakefield will kick m-th bunch by

$$V_{T,\mathrm{LRW}} = a_i S_{i,m} L \tag{23}$$

at i-th cavity, where a_i is the misalignment of the i-th cavity. Assuming $a_i = 400\mu$m, which is estimated tolerance of static cavity offset misalignment,

$$|S_{i,m}| < 1.75 \times 10^5 V/m^2 \tag{24}$$

is required. For the same assumption as in the previous section (With damping), for large m,

$$|S_{i,m}| \approx \frac{qA_0}{\sqrt{2}} \frac{1}{2\alpha t_b}, \tag{25}$$

then the requirement becaomes

$$\frac{A_0}{\alpha t_b} < 1.55 \times 10^{14} V/C/m^2. \tag{26}$$

III. EXAMPLE - HIGHER ORDER MODES DESCRIBED IN TESLA-TDR

For example, the long-range wakefunction and 'sum-wake' calculated from Higher Order Modes (HOM) in TESLA-TDR is shown in Fig. 1, assuming no frequency spread (red circles) and 0.1% frequency spread (blue rectangulars). Only bunch number up to 200 are shown, because the wake is almost completely damped in 200 times bunch spacing. Rms of 'sum-wake' are calculated as in the Table II. From the figures and the table, compared with the Eqs. (15) and (24), the wakefunction with frequency spread of 0.1% satisfies the requirement. On the other hand, the wakefunction without frequency spread will cause some emittance dilution in the head part of a bunch train.

TABLE II: Rms of ‘sum-wake’ for HOM in TESLA-TDR.

Number of Bunches	Frequenncy spread (%)	Rms of ‘sum-wake’ (V/m^2)
2820	0	1.2×10^4
2820	0.1	3.8×10^3
100	0	6.1×10^4
100	0.1	1.9×10^4

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